○途中式は消さないこと　○最後まで計算をすること

問1　次の値を求めよ.

(1) $a^2 \times a^5$

Ans. a^7

(2) $(a^3)^5$

Ans. a^{15}

(3) $(3a^3b^2)^3$

Ans. $27a^9b^6$

(4) $(2a^2)^4 \times (ab^2)^3$

$= 16a^8 \times a^3b^6$

Ans. $16a^{11}b^6$

(5) $a^8 \div a^5$

Ans. a^3

(6) $a^6b^5 \div (a^2b)^2$

$a^6b^5 \div a^4b^2$

Ans. a^2b^3

(7) $3^7 \times 3^3 \div 3^9$

$= 3^{7+3-9} = 3$

Ans. 3

(8) $(5^2 \times 3^3)^3 \div 5^5 \div 3^7$

$5^6 \times 3^9 \div 5^5 \div 3^7$

$= (5^6 \div 5^5) \times (3^9 \div 3^7) = 5 \times 3^2$

$= 5 \times 9 = 45$

Ans. 45

問2　次の値を求めよ.

(1) 6^0

Ans. 1

(2) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Ans. $\frac{1}{8}$

(3) 7^{-1}

Ans. $\frac{1}{7}$

(4) 10^{-2}

Ans. $\frac{1}{100}$

(5) 5^{-4}

Ans. $\frac{1}{625}$

(6) 1000^0

Ans. 1

問3　$a \neq 0,\ b \neq 0$ のとき，次の式を簡単にせよ.

(1) $a^7 \times a^{-3}$

Ans. a^4

(2) $a^{-8} \times a^6$

Ans. a^{-2}

(3) $\frac{a}{a^4}$

Ans. $\frac{1}{a^3}\left(=a^{-3}\right)$

(4) $\frac{a^2}{a^{-2}} = a^2 \div a^{-2} = a^{2-(-2)} = a^4$

Ans. a^4

(5) $(a^{-3})^2$

Ans. a^{-6}

(6) $(a^{-4})^{-3}$

Ans. a^{12}

(7) $(a^{-2}b)^{-3}$

$= a^6b^{-3}$

Ans. $a^6b^{-3}\left(=\frac{a^6}{b^3}\right)$

(8) $(a^2b^{-2})^4 \times a^{-6} \times b^5$

$= a^8b^{-8} \times a^{-6} \times b^5$

$= a^2b^{-3}$

Ans. $a^2b^{-3}\left(=\frac{a^2}{b^3}\right)$

問4　次の式を簡単にせよ.

(1) $\sqrt[3]{64} = \sqrt[3]{4^3} = 4$

(別) $(64)^{\frac{1}{3}} = (2^6)^{\frac{1}{3}} = 2^2 = 4$

Ans. 4

(2) $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

(別) $(2^4)^{\frac{1}{4}} = 2$

Ans. 2

(3) $\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

(別) $(5^3)^{\frac{1}{3}} = 5$

Ans. 5

(4) $\sqrt[5]{1}$

Ans. 1

(5) $\sqrt[3]{\left(\frac{1}{27}\right)} = \sqrt[3]{\left(\frac{1}{3}\right)^3} = \frac{1}{3}$

(別) $\left(\left(\frac{1}{3}\right)^3\right)^{\frac{1}{3}} = \frac{1}{3}$

Ans. $\frac{1}{3}$

(6) $\sqrt[5]{\left(\frac{32}{243}\right)} \quad \sqrt[5]{\left(\frac{2}{3}\right)^5} = \frac{2}{3}$

(別) $\left[\left(\frac{2}{3}\right)^5\right]^{\frac{1}{5}} = \frac{2}{3}$

Ans. $\frac{2}{3}$

問5　次の計算をせよ．

(1) $\sqrt[4]{5}\times\sqrt[4]{125}=\sqrt[4]{5\times125}=\sqrt[4]{5^4}=5$

(別) $(5)^{\frac{1}{4}}\times(5^3)^{\frac{1}{4}}=5^{\frac{4}{4}}=5$

Ans. 5

(2) $\sqrt[3]{2}\times\sqrt[3]{32}=\sqrt[3]{2\times32}=\sqrt[3]{4^3}=4$

(別) $2^{\frac{1}{3}}\times32^{\frac{1}{3}}=2^{\frac{1}{3}}\times2^{\frac{5}{3}}=2^{\frac{6}{3}}=2^2=4$

Ans. 4

(3) $\dfrac{\sqrt[3]{32}}{\sqrt[3]{4}}=\sqrt[3]{\dfrac{32}{4}}=\sqrt[3]{8}=\sqrt[3]{2^3}=2$

(別) $32^{\frac{1}{3}}\div4^{\frac{1}{3}}=2^{\frac{5}{3}}\div2^{\frac{2}{3}}=2^1=2$

Ans. 2

(4) $\dfrac{\sqrt[4]{64}}{\sqrt[4]{4}}=\sqrt[4]{\dfrac{64}{4}}=\sqrt[4]{16}=\sqrt[4]{2^4}=2$

(別) $(64)^{\frac{1}{4}}\div4^{\frac{1}{4}}=2^{\frac{6}{4}}\div2^{\frac{2}{4}}=2^{\frac{4}{4}}=2^1$

Ans. 2

(5) $(\sqrt[4]{9})^2=(\sqrt[4]{3^2})^2=\sqrt[4]{3^4}=3$

(別) $(9^{\frac{1}{4}})^2=9^{\frac{1}{2}}=(3^2)^{\frac{1}{2}}=3$

Ans. 3

(6) $(\sqrt[3]{2})^9=(\sqrt[3]{2^3})^3=2^3=8$

(別) $(2^{\frac{1}{3}})^9=2^3=8$

Ans. 8

(7) $(\sqrt{5^2})^3=5^3=125$

(別) $\{(5^{\frac{2}{2}})^{\frac{1}{2}}\}^3=5^3=125$

Ans. 125

(8) $\sqrt[3]{\sqrt[5]{3}}=\sqrt[15]{3}$

(別) $\sqrt[3]{3^{\frac{1}{5}}}=(3^{\frac{1}{5}})^{\frac{1}{3}}=3^{\frac{1}{15}}=\sqrt[15]{3}$

Ans. $\sqrt[15]{3}$

(9) $\sqrt[3]{\sqrt{2}}=\sqrt[6]{2}$

(別) $\sqrt[3]{2^{\frac{1}{2}}}=(2^{\frac{1}{2}})^{\frac{1}{3}}=2^{\frac{1}{6}}=\sqrt[6]{2}$

Ans. $\sqrt[6]{2}$

(8) $\sqrt[4]{\sqrt{256}}=\sqrt[8]{256}=\sqrt[8]{2^8}=2$

(別) $\sqrt[4]{256^{\frac{1}{2}}}=(256^{\frac{1}{2}})^{\frac{1}{4}}=256^{\frac{1}{8}}=(2^8)^{\frac{1}{8}}=2$

Ans. 2

問6　次の数を簡単にせよ．

(1) $36^{\frac{1}{2}}=(6^2)^{\frac{1}{2}}=6$

Ans. 6

(2) $27^{\frac{1}{3}}=(3^3)^{\frac{1}{3}}=3$

Ans. 3

(3) $81^{\frac{3}{4}}=(3^4)^{\frac{3}{4}}=3^3=27$

Ans. 27

(4) $32^{-\frac{1}{5}}=(2^5)^{-\frac{1}{5}}=2^{-1}=\dfrac{1}{2}$

Ans. $\dfrac{1}{2}$

(5) $64^{-\frac{2}{3}}=(2^6)^{-\frac{2}{3}}=2^{-4}=\dfrac{1}{16}$

Ans. $\dfrac{1}{16}$

(6) $25^{-\frac{3}{2}}=(5^2)^{-\frac{3}{2}}=5^{-3}=\dfrac{1}{125}$

Ans. $\dfrac{1}{125}$

問7　次の式を指数の形 $a^{\frac{n}{m}}$ に直しなさい．ただし，a は小さい整数になるようにせよ．

(1) $\sqrt[3]{2}=2^{\frac{1}{3}}$

Ans. $2^{\frac{1}{3}}$

(2) $\sqrt{27^3}=\sqrt{(3^3)^3}=\sqrt{3^9}=3^{\frac{9}{2}}$

Ans. $3^{\frac{9}{2}}$

(3) $\dfrac{1}{\sqrt[4]{5}}=\dfrac{1}{5^{\frac{1}{4}}}=5^{-\frac{1}{4}}$

Ans. $5^{-\frac{1}{4}}$

(4) $\dfrac{1}{\sqrt[3]{16}}=\dfrac{1}{(2^4)^{\frac{1}{3}}}=\dfrac{1}{2^{\frac{4}{3}}}=2^{-\frac{4}{3}}$

Ans. $2^{-\frac{4}{3}}$

問8　次の式を計算せよ．

(1) $3^{\frac{1}{2}} \times 3^{\frac{3}{2}} = 3^{\frac{4}{2}} = 3^2 = 9$

Ans. 9

(2) $2^{\frac{1}{2}} \times 2^{-\frac{5}{2}} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Ans. $\frac{1}{4}$

(3) $\left(5^{\frac{3}{4}}\right)^{\frac{8}{3}} = 5^{\frac{3}{4}\times\frac{8}{3}} = 5^2 = 25$

Ans. 25

(4) $\left(49^{-\frac{2}{3}}\right)^{\frac{3}{4}} = 49^{-\frac{2}{3}\times\frac{3}{4}} = 49^{-\frac{1}{2}} = (7^2)^{-\frac{1}{2}} = 7^{-1} = \frac{1}{7}$

Ans. $\frac{1}{7}$

(5) $\left(2^{\frac{2}{3}}3^{\frac{1}{2}}\right)^6 = 2^{\frac{2}{3}\times6} \times 3^{\frac{1}{2}\times6} = 2^4 \times 3^3 = 16\times27$

Ans. 432

(6) $5^{\frac{4}{3}} \div 5^{\frac{1}{3}} = 5^{\frac{4}{3}-\frac{1}{3}} = 5^1 = 5$

Ans. 5

(7) $27^{\frac{1}{2}} \div 27^{\frac{1}{6}} = 27^{\frac{1}{2}-\frac{1}{6}} = 27^{\frac{3}{6}-\frac{1}{6}} = 27^{\frac{2}{6}} = 27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3$

Ans. 3

(8) $6^{-\frac{5}{2}} \div 6^{-\frac{1}{2}} = 6^{-\frac{5}{2}-(-\frac{1}{2})} = 6^{-\frac{4}{2}} = 6^{-2} = \frac{1}{36}$

Ans. $\frac{1}{36}$

問9　次の式を計算せよ．

(1) $3^{\frac{1}{4}} \times 3^{\frac{1}{2}} \times 3^{\frac{5}{4}} = 3^{\frac{1}{4}+\frac{1}{2}+\frac{5}{4}} = 3^{\frac{4+2+5}{4}} = 3^{\frac{11}{4}} = 9$

Ans. 9

(2) $5^{\frac{1}{4}} \times 5^{\frac{1}{3}} \div 5^{\frac{1}{12}} = 5^{\frac{1}{4}+\frac{1}{3}-\frac{1}{12}} = 5^{\frac{3+4-1}{12}} = 5^{\frac{6}{12}} = 5^{\frac{1}{2}}$

Ans. $5^{\frac{1}{2}}$

(3) $2^{\frac{1}{2}} \div 2^{\frac{7}{10}} \times 2^{\frac{1}{5}} = 2^{\frac{1}{2}-\frac{7}{10}+\frac{1}{5}} = 2^{\frac{5-7+2}{10}} = 2^0 = 1$

Ans. 1

(4) $\sqrt[3]{7} \times \sqrt{7} \times \sqrt[6]{7} = 7^{\frac{1}{3}} \times 7^{\frac{1}{2}} \times 7^{\frac{1}{6}} = 7^{\frac{2+3+1}{6}} = 7^1 = 7$

Ans. 7

(5) $\sqrt[3]{3^2} \times \sqrt{3} \div \sqrt[6]{3} = 3^{\frac{2}{3}} \times 3^{\frac{1}{2}} \times 3^{-\frac{1}{6}} = 3^{\frac{4+3-1}{6}} = 3^1 = 3$

Ans. 3

(6) $\sqrt{6} \div \sqrt[6]{6} \times \sqrt[3]{6^5} = 6^{\frac{1}{2}} \div 6^{\frac{1}{6}} \times 6^{\frac{5}{3}} = 6^{\frac{3-1+10}{6}} = 6^2 = 36$

Ans. 36

(7) $\sqrt[4]{10} \times \sqrt[4]{8} = \sqrt[4]{80} = \sqrt[4]{2^4 5} = 2\sqrt[4]{5}$

(別) $10^{\frac{1}{4}} \times 8^{\frac{1}{4}} = 2^{\frac{1}{4}}5^{\frac{1}{4}} \times 2^{\frac{3}{4}} = 2^{\frac{4}{4}}5^{\frac{1}{4}} = 2\sqrt[4]{5}$

Ans. $2\sqrt[4]{5}$

(8) $\sqrt[3]{20} \times \sqrt[3]{50} = \sqrt[3]{20\times50} = \sqrt[3]{1000} = \sqrt[3]{10^3} = 10$

(別) $20^{\frac{1}{3}} \times 50^{\frac{1}{3}} = (20\times50)^{\frac{1}{3}} = (1000)^{\frac{1}{3}} = (10^3)^{\frac{1}{3}} = 10$

Ans. 10

(9) $4^{\frac{1}{3}} \times 8^{\frac{1}{6}} \div 2^{\frac{1}{6}} = 2^{\frac{2}{3}} \times 2^{\frac{3}{6}} \div 2^{\frac{1}{6}} = 2^{\frac{4+3-1}{6}} = 2^1 = 2$

Ans. 2

(10) $\sqrt[3]{2} \times \sqrt[6]{2} \div \sqrt[4]{4} = 2^{\frac{1}{3}} \times 2^{\frac{1}{6}} \times 2^{-\frac{2}{4}} = 2^{\frac{4+2+6}{12}} = 2^1$

Ans. 2

(11) $\sqrt[4]{27} \times \sqrt{27} \div \sqrt[4]{3} = 3^{\frac{3}{4}} \times 3^{\frac{3}{2}} \div 3^{\frac{1}{4}} = 3^{\frac{3+6-1}{4}} = 3^{\frac{8}{4}} = 9$

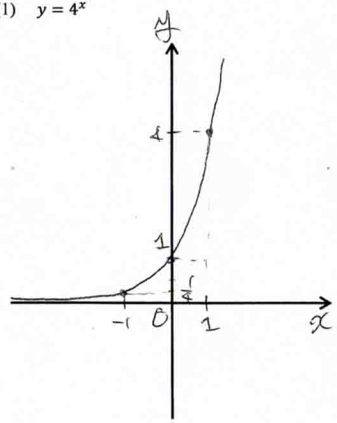
Ans. 9

(12) $\sqrt{27} \times \sqrt[3]{81} \div \sqrt[6]{243} = 3^{\frac{3}{2}} \times 3^{\frac{4}{3}} \div 3^{\frac{5}{6}} = 3^{\frac{9+8-5}{6}} = 3^2 = 9$

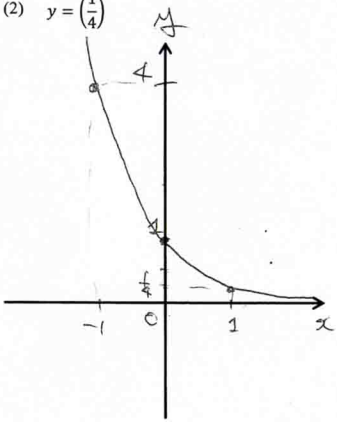
Ans. 9

問1 次のグラフを書きなさい.

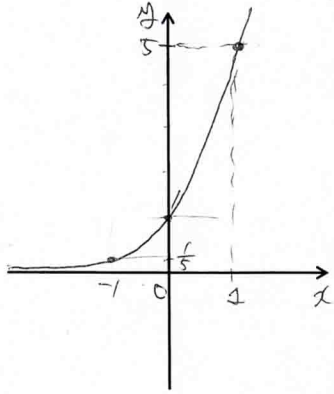
(1) $y = 4^x$



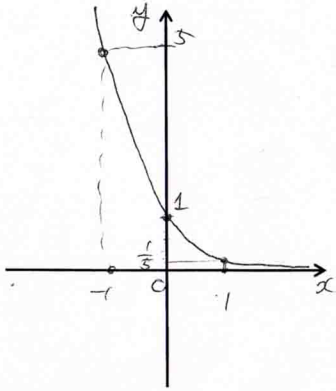
(2) $y = \left(\frac{1}{4}\right)^x$



(3) $y = 5^x$



(4) $y = \left(\frac{1}{5}\right)^x$



問2 次の数を小さい順に並べよ.

(1) $2^3,\ 2^{-1},\ 2^{\frac{1}{2}},\ 2^{-2}$

底が $2>1$ なので.

$2^{-2} < 2^{-1} < 2^{\frac{1}{2}} < 2^3$

Ans. $2^{-2} < 2^{-1} < 2^{\frac{1}{2}} < 2^3$

(2) $3^2,\ 3^{-\frac{3}{2}},\ 1,\ 3^{-1}$

底が $3>1$ なので

$3^{-\frac{3}{2}} < 3^{-1} < 1 < 3^2$

Ans. $3^{-\frac{3}{2}} < 3^{-1} < 1 < 3^2$

(3) $\left(\frac{9}{10}\right)^3,\ \left(\frac{9}{10}\right)^{-3},\ 1,\ \left(\frac{9}{10}\right)^2$

底が $0<\frac{9}{10}<1$ なので

$\left(\frac{9}{10}\right)^3 < \left(\frac{9}{10}\right)^2 < 1 < \left(\frac{9}{10}\right)^{-3}$

Ans. $\left(\frac{9}{10}\right)^3 < \left(\frac{9}{10}\right)^2 < 1 < \left(\frac{9}{10}\right)^{-3}$

(4) $\left(\frac{1}{5}\right)^5,\ \frac{1}{5},\ \left(\frac{1}{5}\right)^2,\ \left(\frac{1}{5}\right)^{-\frac{1}{2}}$

底が $0<\frac{1}{5}<1$ なので

$\left(\frac{1}{5}\right)^5 < \left(\frac{1}{5}\right)^2 < \frac{1}{5} < \left(\frac{1}{5}\right)^{-\frac{1}{2}}$

Ans. $\left(\frac{1}{5}\right)^5 < \left(\frac{1}{5}\right)^2 < \frac{1}{5} < \left(\frac{1}{5}\right)^{-\frac{1}{2}}$

(5) $3,\ \sqrt{\frac{1}{3}},\ \sqrt[3]{3},\ \sqrt[4]{27}$

$3^{-\frac{1}{2}},\ 3^{\frac{1}{3}},\ 3^{\frac{3}{4}}$

底が $3>1$ なので

$3^{-\frac{1}{2}} < 3^{\frac{1}{3}} < 3^{\frac{3}{4}} < 3$

Ans. $\sqrt{\frac{1}{3}} < \sqrt[3]{3} < \sqrt[4]{27} < 3$

(6) $\sqrt{6},\ \sqrt[4]{216},\ 1,\ \sqrt[3]{36}$

$6^{\frac{1}{2}},\ 6^{\frac{3}{4}},\ 6^0,\ 6^{\frac{2}{3}}$

底が $6>1$ なので

$6^0 < 6^{\frac{1}{2}} < 6^{\frac{2}{3}} < 6^{\frac{3}{4}}$

Ans. $1 < \sqrt{6} < \sqrt[3]{36} < \sqrt[4]{216}$

(7) $3,\ 1,\ 3^{-1},\ \sqrt{3}$

$3^0,\ 3^{\frac{1}{2}}$

底が $3>1$ なので

$3^{-1} < 3^0 < 3^{\frac{1}{2}} < 3^1$

Ans. $3^{-1} < 1 < \sqrt{3} < 3$

(8) $(0.7)^{-3},\ 1,\ 0.7,\ (0.7)^{\frac{1}{2}}$

底が $0<0.7<1$ なので

$0.7^1 < (0.7)^{\frac{1}{2}} < (0.7)^0 < (0.7)^{-3}$

1

Ans. $0.7 < (0.7)^{\frac{1}{2}} < 1 < (0.7)^{-3}$

問12 次の方程式・不等式を解け.

(1) $2^x = 8$

$2^x = 2^3$

$x = 3$

Ans. $x = 3$

(2) $4^x = 32$

$2^{2x} = 2^5$

$2x = 5$

$x = \frac{5}{2}$

Ans. $x = \frac{5}{2}$

(3) $3^x = \frac{1}{81}$

$3^x = 3^{-4}$

$x = -4$

Ans. $x = -4$

(4) $\left(\frac{1}{25}\right)^x = \frac{1}{5}$

$(5^{-2})^x = 5^{-1}$

$5^{-2x} = 5^{-1}$

$-2x = -1$　$x = \frac{1}{2}$

Ans. $x = \frac{1}{2}$

(5) $3^{2x+1} = 27$

$3^{2x+1} = 3^3$

$2x + 1 = 3$

$2x = 2$

$x = 1$

Ans. $x = 1$

(6) $5^{x-1} = \left(\frac{1}{25}\right)^{2x}$

$5^{x-1} = (5^{-2})^{2x}$

$5^{x-1} = 5^{-4x}$

$x - 1 = -4x$

$5x = 1$　$x = \frac{1}{5}$

Ans. $x = \frac{1}{5}$

(7) $8^x = 4^{2x+3}$

$2^{3x} = 2^{2(2x+3)}$

$3x = 2(2x+3)$

$3x = 4x + 6$

$-x = 6$

$x = -6$

Ans. $x = -6$

(8) $9^x = 3^{1-x}$

$3^{2x} = 3^{1-x}$

$2x = 1 - x$

$3x = 1$

$x = \frac{1}{3}$

Ans. $x = \frac{1}{3}$

(9) $2^x \leqq 16$

$2^x \leqq 2^4$

底が2>1より

$x \leqq 4$

Ans. $x \leqq 4$

(10) $9^x > 27$

$3^{2x} > 3^3$

底が3>1より

$2x > 3$

$x > \frac{3}{2}$

Ans. $x > \frac{3}{2}$

(11) $3^x < \frac{1}{27}$

$3^x < 3^{-3}$

底が3>1より

$x < -3$

Ans. $x < -3$

(12) $\left(\frac{1}{4}\right)^x \geqq \frac{1}{128}$

$2^{-2x} \geqq 2^{-7}$

底が2>1より

$-2x \geqq -7$

$x \leqq \frac{7}{2}$

Ans. $x \leqq \frac{7}{2}$

(13) $2^{2-x} > 4^x$

$2^{2-x} > 2^{2x}$

底が2>1より

$2 - x > 2x$

$-3x > -2$

$x < \frac{2}{3}$

Ans. $x < \frac{2}{3}$

(14) $\left(\frac{1}{8}\right)^x \geqq \left(\frac{1}{2}\right)^{x+1}$

$\Rightarrow 2^{-3x} \geqq 2^{-x-1}$

底が2>1より

$-3x \geqq -x - 1$

$-2x \geqq -1$

$x \leqq \frac{1}{2}$

Ans. $x \leqq \frac{1}{2}$

(15) $\left(\frac{1}{27}\right)^{x-1} \leqq 9^{x-3}$

$3^{-3x+3} \leqq 3^{2(x-3)}$

底が3>1より

$-3x + 3 \leqq 2x - 6$

$-5x \leqq -9$

$x \geqq \frac{9}{5}$

Ans. $x \geqq \frac{9}{5}$

(16) $(0.5)^{3x} > (\sqrt{2})^{x-7}$

$\left(\frac{1}{2}\right)^{3x} > \left(2^{\frac{1}{2}}\right)^{x-7}$

$2^{-3x} > 2^{\frac{1}{2}(x-7)}$

底が2>1より

$-3x > \frac{1}{2}(x-7)$

$-6x > x - 7$

$-7x > -7$

$x < 1$

Ans. $x < 1$

問13 次の方程式を解け.

(1) $2^{2x} + 2^x - 6 = 0$

$2^x = t$ とおくと

$t^2 + t - 6 = 0$

$(t+3)(t-2) = 0$

$t = 2, -3$

$t > 0$ より

$t = 2$

$2^x = 2$

$x = 1$

Ans. $x = 1$

(2) $4^x - 2^{x+1} - 8 = 0$

$2^x = t$ とおくと

$t^2 - 2t - 8 = 0$

$(t-4)(t+2) = 0$

$t = 4, -2$

$t > 0$ より

$t = 4$

$2^x = 4$

$2^x = 2^2$

$\therefore x = 2$

Ans. $x = 2$